Statistics
in Medicine

Tutorial in Biostatistics

Received: 12 October 2015, Accepted: 5 July 2016 Published online 26 July 2016 in Wiley Online Library

(wileyonlinelibrary.com) DOI: 10.1002/sim.7059

Prevalence odds ratio versus prevalence ratio: choice comes with consequences

Ashutosh R. Tamhane,[a*†] Andrew O. Westfall,[a,b] Greer A. Burkholder[a] and Gary R. Cutter[b]

Odds ratio, risk ratio, and prevalence ratio are some of the measures of association which are often reported in research studies quantifying the relationship between an independent variable and the outcome of interest. There has been much debate on the issue of which measure is appropriate to report depending on the study design. However, the literature on selecting a particular category of the outcome to be modeled and/or change in reference group for categorical independent variables and the effect on statistical significance, although known, is scantly discussed nor published with examples. In this article, we provide an example of a cross-sectional study wherein prevalence ratio was chosen over (Prevalence) odds ratio and demonstrate the analytic implications of the choice of category to be modeled and choice of reference level for independent variables. Copyright © 2016 John Wiley & Sons, Ltd.

Keywords: prevalence odds ratio; prevalence ratio; odds ratio; risk ratio; association

Introduction

Odds ratio (OR) and risk ratio (RR) are two commonly used measures of association reported in research studies. In cross-sectional studies, the OR is also referred to as the prevalence OR (POR) when prevalent cases are included, and, instead of the RR, the prevalence ratio (PR) is calculated. However, it should be noted that, although, mathematical calculations are the same, there are inherent differences in ORs for each study design. Similarly, PR as such neither equals the RR nor the incidence (density) rate ratio.

The literature is abundant with articles discussing advantages/disadvantages of POR/OR versus PR/RR and debate about the 'appropriate' measure of association [1–19]. One of the advantages of OR is that they are preferred for their convenient mathematical property, for example in the Cornfield-chi-square statistics in unstratified analysis, in the Mantel–Haenszel OR in stratified analysis, and in the logistic regression model for multivariable analyses. For dichotomous data with binomial distributions, the log(OR) is considered as a convenient mean for modeling the probability of an outcome when RR have potential of producing estimated probabilities beyond the zero to one range. Furthermore, the log(OR) is directly related to Bayes theorem and is the natural (time invariant) measure in stochastic-risk modeling. However, Poisson modeling will be appropriate for incidence data leading to Poisson distribution in stable cohorts and proportional hazard modeling in dynamic populations where RR will be more intuitive. Greenland [1] presents a strong theoretical argument against the use of OR and comments that 'only incidence differences and ratios possess direct interpretations as measures of impact on average risk or hazard.' He further comments that ORs are useful only when they serve as incidence-ratio (i.e. RR) estimates and logistic and log-linear models are useful only insofar as they provide improved (smoothed) estimates of incidence differences or ratios. The choice of measure of association also affects assessment of confounding. When confounding is defined using 'collapsibility', RR and not the OR is an intrinsic measure of interest [19].

Similarly, issues of the 'overestimation' of the strength of associations and reciprocity of OR have been extensively addressed in various articles and books [1,5,7,8,10,13,18,20–22]. Note, however, the term 'overestimation' applies when one wishes OR to be an approximation of RR; otherwise, both are

[a] *Department of Medicine, Division of Infectious Diseases, University of Alabama at Birmingham, Birmingham, AL, U.S.A.*
[b] *Department of Biostatistics, University of Alabama at Birmingham, Birmingham, AL, U.S.A.*
**Correspondence to: Ashutosh R Tamhane, Department of Medicine, Division of Infectious Diseases, University of Alabama at Birmingham, Birmingham, AL, U.S.A.*
[†] *E-mail: tamhane@uab.edu*

valid measures of associations estimating different population parameters. The property of reciprocity (changing reference category of a dichotomized variable will yield 'reciprocal' estimates) of OR is also well known.

Although, much of the literature for the above-mentioned properties of OR/RR (and POR/PR) is available, changes in statistical significance (p-value) for PR depending upon the category of the outcome modeled or choice of reference category for categorical independent variables are not commonly discussed. In this article, we provide an example of a cross-sectional study wherein PR was chosen over POR and demonstrate the analytic implications, especially with regard to statistical significance, for each measure of association. These implications could very well be important in the conclusions of various investigations and require careful consideration in planning studies and/or thought about the choice of reference group.

Methods

Research study

The aim of our cross-sectional study was to examine predictors of hypertension (HT) control in a cohort of HIV-positive patients. Overall, the analysis of the study included descriptive statistics with univariate and multivariable analyses examining association of multiple predictors with HT control. However, for the purpose of this article and simplicity, we will discuss results only pertaining to one predictor: Race-Sex combination (White-Male, White-Female, Black-Male, and Black-Female).

Statistical analysis

Analyses were conducted using SAS statistical software (version 9.3, Cary NC). POR and PR were calculated using the PROC GENMOD procedure with binomial distribution and logit or log links, respectively. In situations where convergence problems arise, Poisson regression and (modified) Poisson regression with robust standard errors approaches have been suggested [2,14,23,24]. Although, we did not encounter convergence problems for the specified independent predictor (i.e. Race-Sex), models were also run using Poisson regression with robust standard errors to examine consistency of the results obtained using binomial distribution.

Results

"Overestimation" of strength of association

Of the 699 study participants, 380 (54.4%) had achieved HT control (Table I).

Because of the high prevalence of the outcome, we chose PR over POR, as POR would have 'overestimated' the strength of the association considerably. For example, with Black-Male as the reference category, POR for White-Female was 2.63 (Table IIA) while PR was 1.48 (Table IIB) when (HT) control = 'Yes' was modeled ('No' being the reference group). Likewise, overestimation by POR is evident when control = 'No' was modeled (POR = 0.38 versus PR = 0.56).

When the point estimates for PRs with different reference categories were compared (Tables IIB and IIC), the change was approximately 10%. For example, PR for While-Female changed from 1.48

Table I. Descriptive characteristics by hypertension control status.

Predictor	Hypertension control		Overall
	Yes = 380	No = 319	N = 699
	n (%)*	n (%)*	n
Race-Sex			
White-Female	24 (70.6)	10 (29.4)	34
White-Male	159 (58.9)	111 (41.1)	270
Black-Female	80 (53.3)	70 (46.7)	150
Black-Male	117 (47.8)	128 (52.2)	245

*Row percentages.

Table II. Prevalence odds ratios and 12 prevalence ratios examining association between race-sex and hypertension control.

Predictor	Prevalence odds ratio (A)				Prevalence ratio (B)				Prevalence ratio (C)			
	HT Control = 'Yes' modeled		HT Control = 'No' modeled		HT Control = 'Yes' modeled		HT Control = 'No' modeled		HT Control = 'Yes' modeled		HT Control = 'No' modeled	
	POR (95% CI)	*p*-value	PR (95% CI)	*p*-value	PR (95% CI)	*p*-value	PR (95% CI)	*p*-value	PR (95% CI)	*p*-value	PR (95% CI)	*p*-value
Black-Male	Ref	Ref	Ref	Ref	Ref	Ref	Ref	Ref	0.90 (0.73–1.09)	0.28	1.12 (0.91–1.38)	0.29
White-Female	2.63 (1.20–5.72)	0.02	0.38 (0.18–0.83)	0.02	1.48 (1.15–1.90)	0.003	0.56 (0.33–0.96)	0.04	1.32 (1.02–1.72)	0.04	0.63 (0.36–1.09)	0.10
White-Male	1.57 (1.11–2.22)	0.01	0.64 (0.45–0.90)	0.01	1.23 (1.05–1.45)	0.01	0.79 (0.65–0.95)	0.01	1.10 (0.92–1.32)	0.28	0.88 (0.71–1.10)	0.26
Black-Female	1.25 (0.83–1.88)	0.28	0.80 (0.53–1.20)	0.28	1.12 (0.92–1.36)	0.28	0.89 (0.72–1.10)	0.29	Ref	Ref	Ref	Ref

CI, confidence interval; HT, hypertension; POR, prevalence odds ratio; PR, prevalence ratio; Ref, reference category.

Table A: PORs are reciprocals of each other and *p*-values are the same regardless of which outcome (yes or no) is modeled.

Table B: PRs are NOT reciprocals of each other and *p*-values are NOT the same thus choice of outcome category modeled matters. Also, compared with PORs in Table A, PRs are closer to the null value of 1.

Table C: Compared with PRs in Table B, the *p*-values for the white-female and white-male changed considerably.

(Table IIB) to 1.32 (Table IIC) when reference changed from Black-Male to Black-Female, respectively. Similar change was observed for White-Males (PR = 1.23 versus PR = 1.10).

Property of reciprocity

When PORs were compared for outcome = 'Yes' versus 'No', as expected, they were reciprocals of each other [e.g. White-Female: Yes = 2.63 and No = 0.38 (=1/2.63)] (Table IIA). Again as expected, this reciprocity was not observed for PRs (PR: Yes = 1.48 versus No = 0.56) (Table IIB).

Statistical significance: p-value

The *p*-values remained exactly the same for PORs irrespective of whether the outcome = 'Yes' or 'No' was modeled (e.g. White-Female: $p = 0.02$) (Table IIA). In contrast, the *p*-values changed considerably for PRs depending upon the outcome modeled (e.g. White-Female: Yes = 0.003 versus No = 0.04) (Table IIB). Such a change was especially seen for While-Females, but not so for White-Males and Black-Females. Furthermore, when the reference category was changed from Black-Males (Table IIB) to Black-Females (Table IIC), the *p*-value for White-Female changed from 0.04 when control = 'Yes' was modeled to 0.10 when control = 'No' was modeled; a change from being statistically significant (at 0.05 level) to not being statistically significant. Similar results were obtained with Poisson regression using robust standard error method; the 95% confidence intervals generated (by Poisson modeling) were the same generated for POR (logit link) and PR (log link) in binomial distribution.

Discussion

'Overestimation' of strength of association

Overestimation of strength of association by OR as compared to RR has been explained in detail in various books [21,22]. To note, however, that term 'overestimation' applies when one wishes OR to be an approximation of RR; otherwise, both are valid measures of associations estimating different population parameters whose use depends on various reasons. The same logic could be applied for a cross-sectional study explaining discrepancy between POR and PR. In brief, as shown in Table III, it is a function of the mathematical formula and is related to the term $\left[1 - \left(\frac{c}{c+d}\right) / 1 - \left(\frac{a}{a+b}\right)\right]$ because of which POR overestimates PR [21,22]. When the proportion of outcome is 'rare' (e.g. <10%), POR and PR are closer to each other. The magnitude of discrepancy between the POR and PR depending upon the incidence/prevalence of the outcome is well presented in figures in Zhang et al. [25] and Schmidt et al. [26] papers published previously. On a side note, although the mathematical computation for ORs in general are the same for various study designs, different values could be obtained because of selection bias related to study designs [16,27].

Table III. General 2 × 2 table for a cross-sectional study.

Predictor	Outcome		Total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
	a + c	b + d	N

Prevalence odds ratio (POR):
a. Outcome = 'Yes': $\left(\frac{ad}{bc}\right)$
b. Outcome = 'No': $\left(\frac{bc}{ad}\right)$ [Note: reciprocal of equation for outcome = 'Yes']
Prevalence ratio (PR):
a. Outcome = 'Yes': $\left(\frac{a}{a+b}\right) / \left(\frac{c}{c+d}\right)$
b. Outcome = 'No': $\left(\frac{b}{a+b}\right) / \left(\frac{d}{c+d}\right)$ [Note: not reciprocal of equation for outcome = 'Yes']

$$\text{POR} = \frac{}{1 - \left(\frac{c}{c+d}\right)} \text{ X PR}$$
$$1 - \left(\frac{a}{a+b}\right)$$

The magnitude of the difference in the point estimates of the PRs for White-Females (1.48 vs. 1.32, Tables IIB and IIC, respectively) and While Males (1.23 vs. 1.10, Tables IIB and IIC, respectively) will depend on the difference between the proportions compared. This could also mirror in the discrepancy of *p*-values, as explained later.

Property of reciprocity

As shown in Table III, for POR, the mathematical terms modeled for outcome = 'Yes' $\left(\frac{ad}{bc}\right)$ versus outcome = 'No' $\left(\frac{bc}{ad}\right)$ are reciprocals of each other. However, for PR, when the outcome = 'Yes' is of interest, the term modeled is $\left[\left(\frac{a}{a+d}\right)/\left(\frac{c}{a+b}\right)\right]$ while when outcome = 'No' is of interest, the term is $\left[\left(\frac{b}{a+d}\right)/\left(\frac{d}{c+b}\right)\right]$. That is, the terms modeled are different, and therefore, the property of reciprocity in not observed for PR. Therefore, interpretation of a comparison based on PR is critically important on whether the positive outcome or its negative complement is modeled as also shown by Eckerman et al. with regard to RR [28].

Statistical significance: p-value

For PORs, the reasons for obtaining the same *p*-value irrespective of whether outcome = 'Yes' or 'No' was modeled are related to the property of reciprocity and the term modeled being symmetric [17]. Because of this property, some researchers prefer POR as the only measure of association that needs to be calculated and the choice between outcome = 'Yes' or outcome = 'No' does not affect the results/decisions [29–31]. On the contrary, the property of reciprocity does not hold true for PR and yet, such conversions are used in many applications, such as cost-effectiveness analyses and meta-analyses where authors convert results into the same direction. The terms modeled are different for 'Yes' and 'No' when PR is calculated (Table III), and therefore *p*-values (statistical significance) obtained need not be the same. In other words, one is estimating a model which is not symmetrical with the coding of the dependent variable unlike that of POR. The issue of symmetry is less important when the outcome is rare [17].

Moreover, the magnitude of discrepancy between the *p*-values depends on the difference between proportions compared. If the two proportions are closer to each other (around 50%), the difference between the two *p*-values would not be too 'dramatic.' For example (Table I), with Black-Males being a reference category (47.8% had HT control = 'Yes'), the difference between the proportions of White-Females having HT control (70.6%) was larger than that for White-Males (58.9%) and Black-Females (53.3%). Therefore, the change in *p*-value was more 'dramatic' for White-Females while it remained the same for White-Males and Black-Females.

Conclusions

In this example cross-sectional study reporting PR was deemed more appropriate than reporting POR because of considerable 'overestimation' of the strength of the association by POR. Although, the direction/trend of the association remained the same, the statistical significance of the results did change when reference category for the outcome and/or independent variable was switched while calculating PRs. Therefore, researchers should be cautious of the lack of reciprocity and potentially altered *p*-values for PRs. The study results and the discussion do apply for OR versus RR too. Furthermore, they could be generalized to any disease (acute or chronic) where both POR and PR could be calculated. However, duration of (outcome) disease (and exposure too) does dictate the study design, therefore, the measure of association preferred.

It is worthy of note that the decision should not be based solely on statistical significance, but also on clinical significance, which sometimes is overlooked and undue importance to statistical significance, especially *p*-value is given (e.g. selecting variables solely on *p*-value in univariate analysis to be included in multivariable analysis). We, however, acknowledge that the results of this study may not be applicable to all studies where prevalence of the outcome is high as the results, in particular statistical significance, would vary depending upon the sample size.

Acknowledgements

This research was supported by the University of Alabama at Birmingham Center for AIDS Research an NIH funded program (P30 AI027767) that was made possible by the following institutes: NIAID, NCI, NICHD, NHLBI, NIDA, NIMH, NIA, FIC, and OAR. Dr. Greer Burkholder is funded by NHLBI grant K23HL126570. We would like to thank Dr. Paul Allison (University of Pennsylvania) and Dr. Gerald McGwin (University of Alabama at Birmingham) for their valuable comments.

References

1. Greenland S. Interpretation and choice of effect measures in epidemiologic analysis. *American Journal of Epidemiology* 1987; **125**:761–768.
2. Zocchetti C, Consonni D, Bertazzi P. Estimation of prevalence rate ratios from cross-sectional data. *International Journal of Epidemiology* 1995; **24**:1064–1065.
3. Axelson O, Fredriksson M, Ekberg K. Use of the prevalence ratio v the prevalence odds ratio in view of confounding in cross sectional studies. *Occupational and Environmental Medicine* 1995; **52**:494–496.
4. Hughes K. Odds ratios in cross-sectional studies. *International Journal of Epidemiology* 1995; **24**:463–464.
5. Lee J. Odds ratio or relative risk for cross-sectional data. *International Journal of Epidemiology* 1994; **23**:201–203.
6. Lee J. Estimation of prevalence rate ratios from cross sectional data: a reply. *International Journal of Epidemiology* 1995; **24**:1066–1067.
7. Lee J, Chia K. Estimation of prevalence rate ratios for cross-sectional data: an example in occupational epidemiology. *British Journal of Industrial Medicine* 1993; **50**:861–862.
8. Lee J, Chia K. Prevalence odds ratio v prevalence ratio—a response. *Occupational and Environmental Medicine* 1995; **52**:781–782.
9. Lee L, Chia K. Use of the prevalence ratio v the prevalence odds ratio as a measure of risk in cross sectional studies. *Occupational and Environmental Medicine* 1994; **51**:841.
10. Nurminen M. To use or not to use the odds ratio in epidemiologic analysis? *European Journal of Epidemiology* 1995; **11**:365–371.
11. Osborn J, Cattaruzza M. Odds ratio and relative risk for cross-sectional data. *International Journal of Epidemiology* 1995; **24**:464–465.
12. Stromberg U. Prevalence odds ratio v prevalence ratio. *Occupational and Environmental Medicine* 1994; **51**:143–144.
13. Stromberg U. Prevalence odds ratio v prevalence ratio—some further comments. *Occupational and Environmental Medicine* 1995; **42**:143.
14. Thompson M, Myers J, Kriebel D. Prevalence odds ratio or prevalence ratio in the analysis of cross sectional data: what is to be done? *Occupational and Environmental Medicine* 1998; **55**:272–277.
15. Zocchetti C, Consonni D, Bertazzi P. Relationship between prevalence rate ratios and odds ratios in cross-sectional studies. *International Journal of Epidemiology* 1997; **26**:220–223.
16. Kleinbaum D, Kupper L, Morgenstern H. Epidemiologic Research. Principles and Quantitative Methods: Lifetime Learning Publications: Belmont, California, 1982.
17. Cumminngs P. The relative merits of risk ratios and odds ratios. *Archives of Pediatrics & Adolescent Medicine* 2009; **163**(5):438–445.
18. Walter SD. Choice of effect measure for epidemiological data. *Journal of Clinical Epidemiology* 2000; **53**:931–939.
19. Miettinen OS, Cook EF. Confounding: essence and detection. *American Journal of Epidemiology* 1981; **114**:593–603.
20. Bland JM, Altman D. The odds ratio. *British Medical Journal* 2000; **320**(7247):1468.
21. Szklo M, Nieto FJ. Epidemiology Beyond the Basics (2nd edn). Jones and Bartlett: USA, 2007.
22. Gordis L. Epidemiology (4th edn). Saunders Elsevier: USA, 2004.
23. McNutt L, Wu C, Xue X, Hafner J. Estimating the relative risk in cohort studies and clinical trials of common outcomes. *American Journal of Epidemiology* 2003; **157**(10):940–943.
24. Zou G. A modified poisson regression approach to prospective studies with binary data. *American Journal of Epidemiology* 2004; **159**(7):702–706.
25. Zhang J, Yu K. What's the relative risk? A method of correcting the odds ratio in cohort studies of common outcomes. *Journal of the Americal Medical Association* 1998; **280**(19):1690–1691.
26. Schmidt C, Kohlmann T. When to use the odds ratio or the relative risk? *International Journal of Public Health* 2008; **53**:165–167.
27. Kleinbaum D, Sullivan K, Barker N. A Pocket Guide to Epidemiology. Springer: USA, 2007.
28. Eckerman S, Corry M, Willan A. Indirect comparison: relative risk fallacies and odds solution. *Journal of Clinical Epidemiology* 2009; **62**:1031–1036.
29. Senn S, Walter S, Olkin I. Odds ratio revisited. *Evidence Based Medicine.* 1998; **3**:71.
30. Newman S. Biostatistical Methods in Epidemiology. John Wiley & Sons: New York, NY, 2001.
31. Cook T. Advanced statistics: up with odds ratios! a case for odds ratios when outcomes are common. *Academic Emergency Medicine* 2002; **9**(12):1430–1434.